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*Published in:*

1999 Annual Report - Conference on Electrical Insulation and Dielectric Phenomena, IEEE Publication 99CH36319

*Link to article, DOI:*

[10.1109/CEIDP.1999.804625](https://doi.org/10.1109/CEIDP.1999.804625)

*Publication date:*

1999

*Document Version*

Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

*Citation (APA):*

McAllister, I. W., & Crichton, G. C. (1999). Influence of bulk dielectric polarization and void geometry upon PD transients. In *1999 Annual Report - Conference on Electrical Insulation and Dielectric Phenomena, IEEE Publication 99CH36319* (pp. 197-201). IEEE. <https://doi.org/10.1109/CEIDP.1999.804625>

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1999 Conference on Electrical Insulation and Dielectric Phenomena  
**Influence of Bulk Dielectric Polarization and Void Geometry  
upon PD Transients**

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**Abstract**

The induced charge arising from a partial discharge consists of a component associated with the actual space charge in the void, and one related to changes in the polarization of the bulk dielectric. These changes are brought about by the field produced by the space charge. The influence of the void geometry upon the polarization component of the induced charge is examined for a heterogeneous bulk dielectric system. It is demonstrated that the relative magnitude of this component may either increase or decrease, depending on the ratio of the dielectric permittivities and within which dielectric the void is located. The magnitude of this effect is also dependent upon the prolateness/oblateness of the void geometry and on the orientation of the void.

**Introduction**

A partial discharge in a gaseous void produces a change in the polarization ( $\delta\vec{P}$ ) of the bulk dielectric. This change is reflected in the magnitude of the charge induced upon the detecting electrode [1-3]. Mathematically this induced charge is composed of a component associated with the actual space charge in the void, and one related to changes in the bulk dielectric polarization brought about by the space charge.

In the present paper, the influence of the void geometry upon  $\delta\vec{P}$  is examined

for a heterogeneous bulk dielectric system. It is shown that the component of the induced charge due to  $\delta\vec{P}$  may increase or decrease depending upon the ratio of the dielectric permittivities and within which dielectric the void is located. The magnitude of the effect is also strongly dependent upon the prolateness/oblateness of the void geometry and on the orientation of the void. This behaviour is directly reflected in terms of the Poissonian induced charge, and consequently is manifest in the magnitude of the recorded PD transient [4].

**Polarization Component of the  
Poissonian Induced Charge**

The Poissonian induced charge is that component of the induced charge which is rigidly linked to the space charge source, and which together with this source gives rise to the Basic Poisson Field [2]. The final value of the Poissonian induced charge  $q$  due to a partial discharge may be resolved mathematically into two components:

$$q = q_{\mu} + q_P \quad (1)$$

The component  $q_{\mu}$  is the induced charge directly associated with the space charge in the void;  $q_P$  represents the induced charge related to the change in dielectric polarization ( $\delta\vec{P}$ ) due to the presence of this space charge [2].

With reference to the detecting elect-

rode and the induced charge, the effect of the void wall charges arising from the partial discharge can be considered as the effect of an electric

dipole of moment  $\vec{\mu}$  located within the void [1]. The total Poissonian induced charge arising from this dipole is given by

$$q = -\vec{\mu} \cdot \vec{\nabla} \lambda \quad (2)$$

where  $\lambda$  is the proportionality factor between the charge in the void and the induced charge on the detecting electrode. The  $\lambda$  function, which is a solution of the general Laplace equation [2], inherently takes into account the polarization of the dielectric.

With the proportionality factor,  $\phi$ , it is possible to determine the component of the Poissonian induced charge related to the void space charge alone, viz. we have

$$q_{\mu} = -\vec{\mu} \cdot \vec{\nabla} \phi \quad (3)$$

The  $\phi$  function is a solution of the reduced Laplace's equation [2].

Application of these two functions allows the polarization of the dielectric to be accounted for explicitly: viz., on using (1), (2) and (3), the polarization component  $q_P$  of the Poissonian induced charge may be expressed as

$$q_P = -\vec{\mu} \cdot (\vec{\nabla} \lambda - \vec{\nabla} \phi) \quad (4)$$

### The $\lambda_0$ Function

If the dimensions of the void are such that  $\vec{\nabla} \lambda$  may be assumed constant within the void, then we can introduce another function,  $\lambda_0$ . This function, which is derived for the same boundary conditions but without the void, represents the unperturbed  $\lambda$  function. As both  $\lambda$  and  $\lambda_0$  are solutions of Laplace's equation,

then by mathematical analogy with electrostatic fields, the relationship between the  $\lambda$  and  $\lambda_0$  functions is given by

$$\vec{\nabla} \lambda = h \vec{\nabla} \lambda_0, \quad 1 \leq h \leq \epsilon_r \quad (5)$$

For the type of void under consideration, the parameter  $h$  is a scalar which depends on the void geometry and the relative permittivity  $\epsilon_r$  of the bulk medium.

Following the introduction of  $\lambda_0$ ,  $q_P$  is given by

$$q_P = -\vec{\mu} \cdot (h \vec{\nabla} \lambda_0 - \vec{\nabla} \phi) \quad (6)$$

### Heterogeneous Bulk Dielectric System

The influence of  $\delta \vec{P}$  upon the induced charge will be examined for a void located in a planar heterogeneous dielectric system. With this simple geometry it is possible to illustrate the basic consequences associated with  $\delta \vec{P}$ .

We consider a planar electrode geometry with a two layer dielectric. If in rectangular coordinates, the electrodes are represented by  $z = 0$  and  $z = d$ , then the dielectric interface is taken as  $z = s$ , with  $s < d$ . The permittivity of the upper dielectric is  $\epsilon_2$  for which  $s \leq z \leq d$ , while that of the lower is  $\epsilon_1$ , for which  $0 \leq z \leq s$ . If the lower electrode is used as the detecting electrode, then the boundary conditions for the  $\lambda_0$  function are  $\lambda_0 = 1$  for  $z = 0$  and  $\lambda_0 = 0$  for  $z = d$ .

For this simple geometry, the  $\vec{\nabla} \lambda_0$  expressions can be readily shown to be given by

$$\vec{\nabla} \lambda_{01} = \frac{-\epsilon_2 \vec{e}}{\epsilon_1(d-s) + \epsilon_2 s} \quad (7)$$

and

$$\vec{\nabla}\lambda_{02} = \frac{-\epsilon_1 \vec{e}}{\epsilon_1(d-s) + \epsilon_2 s} \quad (8) \quad \vec{\mu} = \mu \vec{e} \quad (11)$$

where  $\vec{e}$  is a unit vector in the positive  $z$  direction. The  $\lambda_0$  subscripts 1 & 2 refer to the upper and lower regions, respectively; i.e. to the different dielectric regions.

For a homogeneous medium,  $\lambda_0 = \phi$  and thus for a planar system we have

$$\vec{\nabla}\lambda_0 = \vec{\nabla}\phi = -\frac{\vec{e}}{d} \quad (9) \quad \frac{d\phi/dz}{h_n d\lambda_{0n}/dz} \geq 1 \quad (13)$$

It may be noted that both (7) and (8) reduce to this expression for  $\epsilon_1 = \epsilon_2$ .

Hence upon undertaking the vector operations, (10) reduces to

$$\frac{q_{Pn}}{q_n} = 1 - \frac{d\phi/dr}{h_n d\lambda_{0n}/dr} \quad (12)$$

From (12) it is clear that the polarity of  $q_{Pn}/q_n$  is dependent upon whether

$$\frac{d\phi/dz}{h_n d\lambda_{0n}/dz} \geq 1 \quad (13)$$

Using (7), (8) and (9) we obtain for a void in medium 1

$$\frac{q_{P1}}{q_1} = 1 - \frac{\epsilon_1(d-s) + \epsilon_2 s}{\epsilon_2 h_1 d} \quad (14)$$

and for a void in medium 2

$$\frac{q_{P2}}{q_2} = 1 - \frac{\epsilon_1(d-s) + \epsilon_2 s}{\epsilon_1 h_2 d} \quad (15)$$

### Induced Charge Component due to $\delta\vec{P}$

To ensure that the concept of  $h$  and (5) are valid, it will be assumed that the void is more than 10 times its greatest linear dimension from the dielectric interface, such that the  $\vec{\nabla}\lambda$  distribution within the void remains uniform: i.e. the existence of this interface does not perturb  $\vec{\nabla}\lambda$  in the void.

With respect to the component of the induced charge related to  $\delta\vec{P}$ , we have upon combining (2), (5) and (6)

$$\frac{q_{Pn}}{q_n} = \frac{-\vec{\mu} \cdot (h_n \vec{\nabla}\lambda_{0n} - \vec{\nabla}\phi)}{-\vec{\mu} \cdot h_n \vec{\nabla}\lambda_{0n}} \quad (10)$$

where  $q_n$  is the total Poissonian induced charge of the heterogeneous system; the subscript  $n = 1, 2$  indicates within which dielectric medium the void is located.

On account of the planar geometry, the resulting dipole moment will possess only a  $z$ -component. This may be directed either away from or towards the coordinate origin. However, without loss of generality, we select the positive  $z$ -direction; viz.

Apart from permittivity and layer dimensions, both (14) and (15) contain  $h$ . Thus the ratio  $q_{Pn}/q_n$  is also dependent upon the specific void geometry.

### Influence of Void Geometry

The influence of the void geometry upon  $q_{Pn}$  will be studied with reference to spheroidal voids oriented such that both  $\vec{\nabla}\lambda_0$  and the applied field are *normal* to the axis of rotation. For such voids,  $h$  is given by [5]

$$h = \frac{K\epsilon_r}{1 + (K-1)\epsilon_r} \quad (16)$$

where  $\epsilon_r$  is the relative permittivity of the relevant bulk dielectric and  $K$  is a dimensionless parameter arising

from the analysis.

For the present void orientation, we have for an oblate spheroidal void [6]

$$K = \frac{2u^3}{(u^2 + 1)\arctan u - u} \quad (17)$$

The variable  $u$  is given by

$$u = \sqrt{(b/a)^2 - 1} \quad (18)$$

where  $a$  and  $b$  are the semi-axes of the oblate spheroid such that  $b/a > 1$ ; i.e.  $a$  is the axis of rotation.

For a prolate spheroidal void ( $b/a < 1$ ) the associated expression for  $K$  is [6]

$$K = \frac{4v^3}{(1 - v^2)\ln\left(\frac{1-v}{1+v}\right) + 2v} \quad (19)$$

where

$$v = \sqrt{1 - (b/a)^2} \quad (20)$$

and  $a$  is again the axis of rotation. Following the evaluation of the  $K$  values the corresponding  $h$  values can be derived and thereafter inserted into (14) and (15).

The variation of  $q_{P1}/q_1$  with  $a/b$  is shown in Fig.1 for  $\epsilon_r = 4$  and selected values of  $\epsilon_2/\epsilon_1$ . If we take the homogeneous case ( $\epsilon_2/\epsilon_1 = 1$ ) as the reference condition, then from Fig.1 it is evident that, when  $\epsilon_2/\epsilon_1 > 1$ ,  $q_{P1}$  is increased. For the present void orientation, it is the oblate void which exhibits the greater relative effect. For  $\epsilon_2/\epsilon_1 < 1$ , there is a reduction in  $q_{P1}$ . This can be of such a degree that the polarity of  $q_{P1}$  is reversed. In this condition, the oblate void displays the greater reductions, both in relative and absolute terms. Furthermore, a comparison of

Fig.1 with the similar variation for the other principal void orientation, see [7], indicates that the present results are essentially the mirror image of those reported in [7].

From Fig.1, it becomes apparent that for a range of values of the permittivity ratio  $\epsilon_2/\epsilon_1$ , the value of  $q_{P1}/q_1$  may be zero. The actual values depend on the functional form of the *homogeneous* curve, and are consequently subject to void shape. From (14), it can be deduced that for oblate voids these ratios lie in the range  $0.6 < \epsilon_2/\epsilon_1 < 1$  for  $\epsilon_r = 4$  and  $s/d = 0.5$ . The value 0.6 becomes the upper limit if prolate voids are considered.

Finally, the *changes* in permittivity ratio required to bring about this zero condition are greater for the prolate voids than for the oblate voids. Such behaviour is the reverse of that observed with the void axis of rotation *parallel* to both the field direction and  $\vec{\nabla}\lambda_0$ ; see [7].

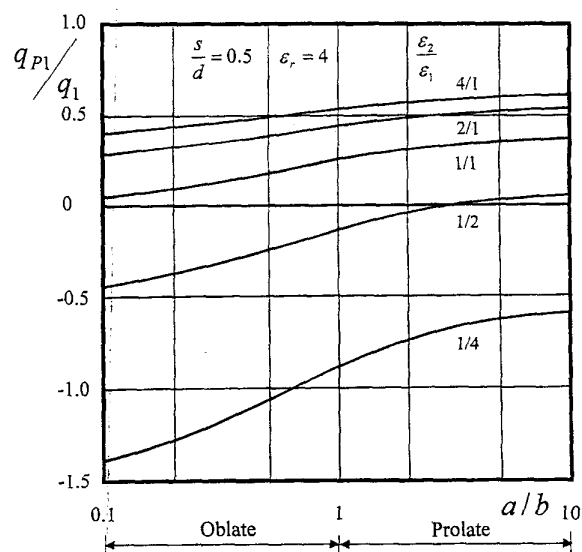


Fig.1. Variation of the polarization component of the Poissonian induced charge for spheroidal voids in a heterogeneous dielectric.

### Conclusions

Changes in dielectric polarization arising from a partial discharge can significantly affect the magnitude of the polarization component of the Poissonian induced charge. For a two dielectric system, this influence is dependent not only upon the ratio of the dielectric permittivities and within which medium the void is located, but also on the void geometry and its orientation. These latter parameters also affect the values of the permittivity ratios associated with a zero polarization component.

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